UK JUNIOR MATHEMATICAL CHALLENGE

TUESDAY 26th APRIL 2005

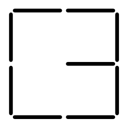
Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **C** $1000 - 100 + 10 - 1 = (1000 - 100) + (10 - 1) = 900 + 9 = 909$.

2. **B** The original figure does not have a line of symmetry, so at least one match must be added. The figure shown, which is created by adding one extra match, does have a line of symmetry, so the smallest number of matches that need to be added is one.

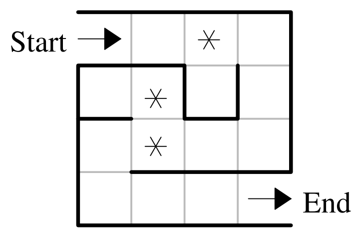


3. **C** As the number of days in a week is odd, Gollum will eat fish on alternate Mondays.

4. **E** Reflected in the mirror, the hands of the clock would have the same appearance at 1.30pm as they would normally have at 10.30pm.

5. **B** As the numbers are all positive, the largest number cannot be the difference between two of the others. Checking all the other options: $1 = 7 - 6$; $6 = 7 - 1$; $5 = 7 - 2$; $2 = 7 - 5$.

6. **D** The diagram shows the squares which Jonny's rat must visit more than once when it goes through the maze.



7. **C** Since one gram is the weight of half a million seeds, 1000 grams (i.e. one kilogram) is the weight of 500 million seeds.

8. **A** Let Reg have x grams of chocolate. Then the ratio of Peg's amount of chocolate to Meg's is $6x : 2x$, that is $3 : 1$.

9. **E** If the sheet of paper had been folded in half once then there would have been two holes in the unfolded sheet. Each additional fold doubles the eventual number of holes in the unfolded sheet so after four folds there will be sixteen holes.

10. **D** When Tilly woke on Thursday morning, she had learned fifteen new words, but had forgotten six of these, so she knew nine words. On Thursday, she learned five new words, so this was the first day on which she reached her target of fourteen words.

11. **A** The mode is the category which contains more than any of the others, so is represented by the largest sector in a pie chart.

12. **E** Remembering that, in the absence of brackets, multiplication is performed before addition, we see that $9 \times 6 + 73 = 54 + 73 = 127$, whereas $96 + 7 \times 3 = 96 + 21 = 117$. It is left as an exercise for the reader to check that the other four calculations are correct.

13. **A** As the squares are equal, the triangle in the figure is isosceles. So its angles are $70°$, $70°$ and $40°$. Hence $x = 360 - (40 + 90 + 90) = 140$.

14. **D** The fractions could be placed in order by writing them all with a common denominator of 630, or by writing them as decimal fractions. However, as they are all close in value to $\frac{1}{2}$, we may consider the value of each fraction minus $\frac{1}{2}$. These are, respectively, 0, $+\frac{1}{6}$, $+\frac{1}{10}$, $+\frac{1}{14}$ and $+\frac{1}{18}$. So, when they are placed in increasing order of size, the fractions are $\frac{1}{2}$, $\frac{5}{9}$, $\frac{4}{7}$, $\frac{3}{5}$ and $\frac{2}{3}$.

15. **A** The sum of the digits of each of the six numbers is 9. This means that they are all multiples of 9, so none of them is prime.

16. **D** $2005 \div 12 = 167$ remainder 1 so if 167 of the larger boxes are used, one bar will remain. If 166 of the larger boxes are used, 13 bars will remain and these cannot fill a whole number of smaller boxes. However, if 165 larger boxes are used, the 25 remaining bars will fill 5 smaller boxes. Using fewer than 165 larger boxes will increase the total number of boxes required since proportionately more of the smaller boxes would be needed, so the required number of boxes is $165 + 5$.

17. **E** There are two right-angled triangles which have their right angle at P: triangles UPQ and UPR. Similarly, there are two right-angled triangles in each case which have their right angle at R, S and U. There are three right-angled triangles which have their right angle at Q: triangles PQT, RQT and UQS. Similarly, there are three right-angled triangles which have their right angle at T, making 14 in all.

18. **A** The subtraction shows the number $10a + b$ subtracted from the number $10b + a$. Their difference is $9b - 9a$. So $9(b - a)$ has unit digit 6 and must therefore have value 36 since this is the only two-digit multiple of 9 which ends in 6. So c is 3. Note that the values of a and b are not unique.
Provided that $b - a$ is 4, the difference of the two numbers will be 36, for example $51 - 15 = 36$; $62 - 26 = 36$; $73 - 37 = 36$ etc.

19. **E** Let the length and breadth of one of the original cards be l and b respectively. Then the lengths of the six sides of the 'L' shape (moving clockwise from the bottom left-hand corner) are l, b, $l - b$, $l - b$, b, l respectively. So $2l + 2b + 2(l - b) = 40$, that is $4l = 40$. We may deduce, therefore, that l is 10, but there is insufficient information for us to find the ratio $b : l$.

20. **B** At most one of the statements is true (as they are mutually contradictory). Indeed, the second statement is true and there is exactly one true statement.

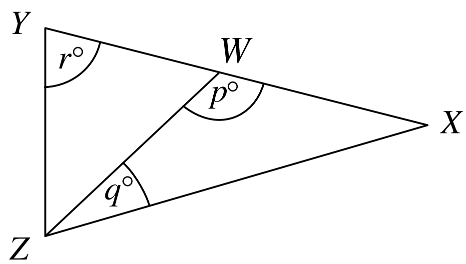
21. D The square labelled A_1 is in the same row as C and D, the same column as E and the same diagonal as B. So it must be A. The square labelled E_2 is the next to be filled in as it is in the same row as A, C and D and in the same diagonal as B. The square labelled B_3 now completes the fourth row. The bottom left-hand corner square must now be A. This is because one of the three remaining squares in the diagonal which runs from bottom left to top right must be A, but the A at the end of row 2 means that A cannot be in either of the last two squares of this diagonal.

	*			A
		B		
D	E_2	C	A_1	B_3
A_4		D_5	E	C_6

The squares D_5 and C_6 may now be filled in. We now see that the square marked * is in the same diagonal as A, B and C and in the same column as E, so it must be D. It is left as an exercise for the reader to complete the grid.

22. C The letters used in TRIANGLE are the same as those used in RECTANGLE, except that C and E have been replaced by I. So the code for TRIANGLE is $9 \times 31\,752\,000 \div (3 \times 5)$, that is 19 051 200.

23. B As $XY = XZ$, $\angle XZY = \angle XYZ$, so $\angle YZW = r° - q°$. In a triangle, an exterior angle is equal to the sum of the two interior opposite angles. Applying this theorem to triangle ZYW:
$\angle ZWX = \angle YZW + \angle ZYW$, that is $p = (r - q) + r$. So $2r = p + q$.



24. C As Jack makes one revolution every five seconds, and Jill one revolution every six seconds, on average they turn through angles of 72° and 60° respectively every second. So, as they are travelling in opposite directions, Jill turns, on average, through an angle of 132° per second relative to Jack. In one minute, therefore, she will turn through an angle of $60 \times 132°$, that is 22 complete revolutions, relative to Jack. So they will pass each other 22 times in the first minute.

25. B Imagine the cross to consist of three horizontal layers: the first layer contains only the cube which was glued to the top face of the original cube. The second layer contains the original cube plus four additional cubes glued to the side faces of that cube. The third layer contains only the cube which was glued to the bottom face of the original cube. When yellow cubes are now added, one cube will be glued to the top face of the blue cube on the top layer and four to its side faces. Eight yellow cubes will be glued to the blue cubes in the second layer and the single blue cube in the third layer will have five yellow cubes glued to it: one to its bottom face and four to its side faces. So, overall, 18 yellow cubes are required.